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Helping Teachers Connect Vocabulary and Conceptual Understanding

A. Susan Gay

Mathematics has many words with specialized meanings, and students encounter these words in texts, on assessments, and as part of class presentations and discussions. Students need to know the meaning of mathematics vocabulary words—whether written or spoken—in order to understand and communicate mathematical ideas (Thompson and Rubenstein 2000).

For better or worse, teachers are role models in their use of mathematics vocabulary. During a class presentation, when they name or describe an object or an action, teachers must have a mastery of vocabulary and use words correctly as they teach. A few years ago, I heard one of my student teachers give the following directions during his algebra 2 lesson presentation: "Graph this expression" and "Evaluate 6^3, 12^4, and n^4 if $n = 3$." These are just two examples of spoken phrases that methods course instructors, student teacher supervisors, and administrators observing in a classroom have heard from new or experienced teachers who struggle at times to use mathematical terminology correctly. This particular observation and my subsequent discussion with the student teacher convinced me that I needed to address specific vocabulary issues beginning with my preservice teachers' methods class. In particular, my preservice teachers needed to be aware of how their use of vocabulary contributes directly to students' understanding or misunderstanding and to learn some vocabulary strategies for use with their future middle and high school students.

The student teacher quoted earlier knew that one does not graph an *expression;* rather, one graphs an *equation.* It is also likely that his algebra 2 students were not confused by his statement; in essence, they "knew what he meant" and were able to construct the graph. However, the distinction between *expression* and *equation* is an important one, and a teacher needs to reinforce the distinction. When questioned, the student teacher responded that *simplify* is a better verb to use when working with 6^3 and 12^4 but that one would *evaluate* n^4 for a given value of n. Yet he was not aware that his lack of clarity could cause confusion for his students about what evaluating a variable expression means.

This article presents a set of class activities I have used in my one-semester methods course for preservice middle and high school mathematics teachers. However, the ideas could be used by any classroom teacher or as part of professional development for in-service teachers.

USING THE CONCEPT ATTAINMENT MODEL OF TEACHING

Vocabulary and its relationship to developing conceptual understanding is the focus of one class period during the semester. I invite a colleague whose specialization is content-area literacy to be a co-presenter for this class period, and he begins the discussion by introducing the concept attainment strategy. Concept attainment is an inductive teaching model developed from the work of Bruner, Goodnow, and Austin (Eggen, Kauchak, and Harder 1979; Joyce, Weil, and Calhoun 2004). In this model, the teacher first selects a concept and pairs of examples and nonexamples. The teacher may or may not choose to tell students the name of the concept. The teacher does present an example and a nonexample and tells students which is which. This first set of illustrations may include more than one pair of an example and a nonexample, thus providing more data for initial comparisons. The students consider the attributes present in both the example and the nonexample and develop a first draft of a definition of the concept. Other pairs are presented, and, with each successive pair, students revise their definition of the concept. Key to the success of the model is the teacher's selection and sequencing of examples and nonexamples so that every example contains all the essential distinguishing attributes of the concept and the nonexamples establish "concept boundaries and limits" (Eggen, Kauchak, and Harder 1979, p. 148). The model's developers note that usually twenty pairs are needed (Joyce, Weil, and Calhoun 2004).

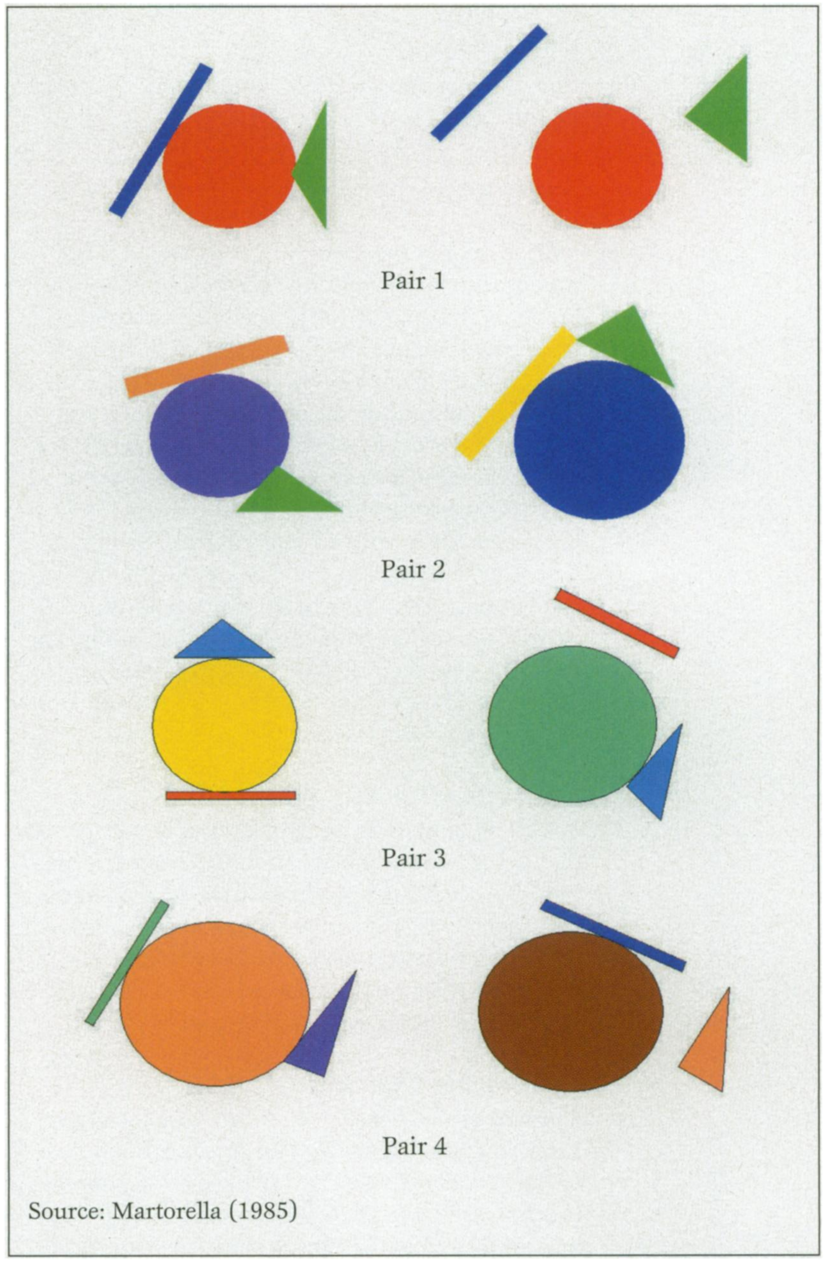

Fig. 1 Examples and nonexamples of a figural (the figural example is on the left, and the nonexample is on the right).

In my class, the preservice teachers are presented with the term *figural,* which we intend as a nonsense word that appears to have some connection to mathematics. Students are then shown pairs of illustrations depicting one example and one nonexample of a figural (see **fig. 1**; the figural examples are on the left, and the nonexamples are on the right). After studying the first pair for about a minute, students are asked for a definition of *figural.* A common first definition is this: "A figural has three shapes that touch." This first definition is written where all students can see it, and everyone has an opportunity to make changes to it. The definition remains in view while the second pair of illustrations is presented. At this point, students

Word	Characteristics
Definition	Example(s)

Fig. 2 Graphic organizer used with preservice teachers

typically revise the definition as follows: "A figural is a circle, triangle, and rectangle that touch." Students continue to refine the term's definition as they view the remaining pairs shown in **figure 1**. A recent group of preservice teachers decided on this final definition: "A figural consists of a rectangle and a triangle that are tangent to a circle but that do not touch each other." Although my colleague is working with a fictional concept, I follow his presentation with an illustration of examples and nonexamples that could be used with middle or high school students to develop a definition of *parallelogram*. We both emphasize that having students develop their own definitions is one way to help them focus on the key characteristics of concepts, thus building conceptual understanding.

TWO VOCABULARY STRATEGIES

We spend most of the remainder of this class period discussing two vocabulary strategies and exploring how they can be incorporated in the mathematics classroom. One strategy makes use of a graphic organizer. Another strategy, called the concept circle, is used in content-area reading classes. Several variations of the concept circle strategy are presented.

The Graphic Organizer

The first strategy makes use of the graphic organizer shown in **figure 2**. This organizer is similar to the verbal and visual word association strategy used in reading (e.g., Readence, Bean, and Baldwin 2001) and to the Frayer model developed as a result of work by Frayer, Fredrick, and Klausmeier (Greenwood 2002).

For a given vocabulary word, this organizer keeps a strong focus on the relationship among the definition of a concept, one or more illustrative examples of the concept, and characteristics of the concept that the word represents. These three sections correspond to Henderson's (1970) three ways of teaching a concept. He noted that when teachers "talk about the properties or characteristics of objects named by a term" (p. 171), they employ the connotative use of the term. When teachers give examples, they use the term in a denotative manner. And when they define the term, they employ the implicative use of the term. Thus, this organizer helps students organize their thinking about a concept in the same three ways that teachers employ to teach the meaning of a concept.

We have found this strategy to be particularly useful when the chosen word is a noun, such as *parabola* or *polygon*. Several examples created by my preservice teachers are presented in **figures 3** and **4**. The creator of the organizer for *counting numbers* (**fig. 3**) had some difficulty listing characteristics other than to say what they are not. The definition is a good one, and the examples are appropriate. The organizer for *parabola* contains a definition with one misspelled word and omission of the condition that a is not equal to zero; a satisfactory list of characteristics; and two appropriate examples. **Figure 4** presents two different organizers for *polygon*. Although the ideas are similar, the second one shows more accurate use of vocabulary, referring to line segments as parts of a polygon

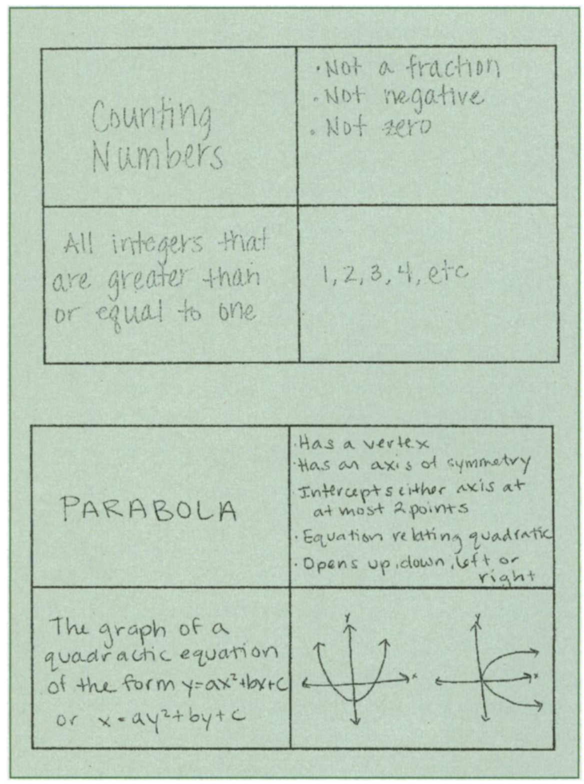

Fig. 3 Vocabulary organizers created by preservice teachers

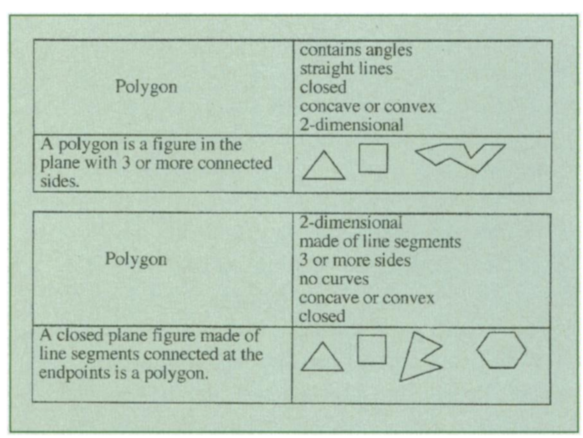

Fig. 4 Vocabulary organizers created by preservice teachers

rather than as straight lines. The definition in the second organizer is more accurate also because the creator stated that the figure must be closed and that the sides must be connected at endpoints.

During this class period, my literacy colleague and I briefly share our previous research with middle and high school students who used this strategy. We note that these middle and high school students were initially better at providing examples than they were at providing definitions or characteristics, but their classroom teachers reported that repeated use of this strategy helped them develop a better conceptual understanding of mathematical words.

The Concept Circle

The second vocabulary strategy is the concept circle, a categorization strategy that encourages students to study words critically, relating them conceptually to one another. The strategy involves writing a word or phrase in each section of a circle and directing students to describe the common attributes or name the relationship that exists among the words or phrases in an attempt to name or label the circle (e.g., Barton and Heidema 2002; Gay and White 2002). The concept circles we present to the methods class have four sections, but other numbers of sections are possible. Two examples created by my preservice teachers are presented in **figure 5**. The concept in the first concept circle could be named *relationships for pairs of angles* or *types of pairs of angles*. *Conic sections* is an appropriate name for the second concept circle.

When we introduce this strategy, we do so using words. So, the first examples created by the preservice teachers also use words. However, part of the mathematics vocabulary is symbolic. These symbols are often used as words to represent various concepts (Rubenstein and Thompson 2001). We tell the preservice teachers that they can create concept circles that use numerals, graphs, equations, and other types of representations. To name or label the concept circle, students must know the meaning of the symbols as well as connect the symbol to the idea it represents and to the written or spoken form (Barton and Heidema 2002). **Figures 6** and **7** include concept circles with symbols in the circle sections.

The concept circle strategy can be modified in various ways. The name or label of the concept circle can be provided, one section of the circle can be blank, and students can be asked to provide another example to complete the concept circle. Or the name or label of the circle can be provided, one nonexample can be included in a section of the circle, and students can be asked to identify the section that is not representative of the concept. An example of each type of modified concept circle created by preservice teachers in my methods class is presented in **figure 6**. For the first modified concept circle, labeled *function*, there are several appropriate responses for the blank section, such as *exponential*, *logarithmic*, or *rational*. This concept circle keeps a focus on the broad categories of functions. For the second modified concept circle, labeled *monomial*,

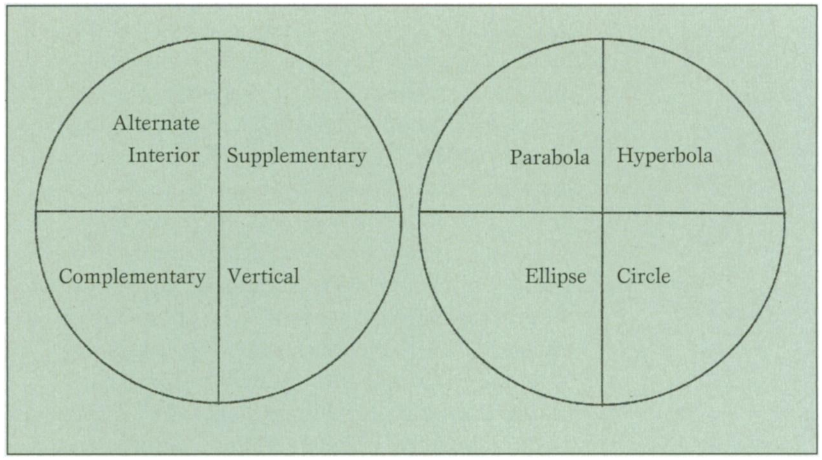

Fig. 5 Concept circles created by preservice teachers

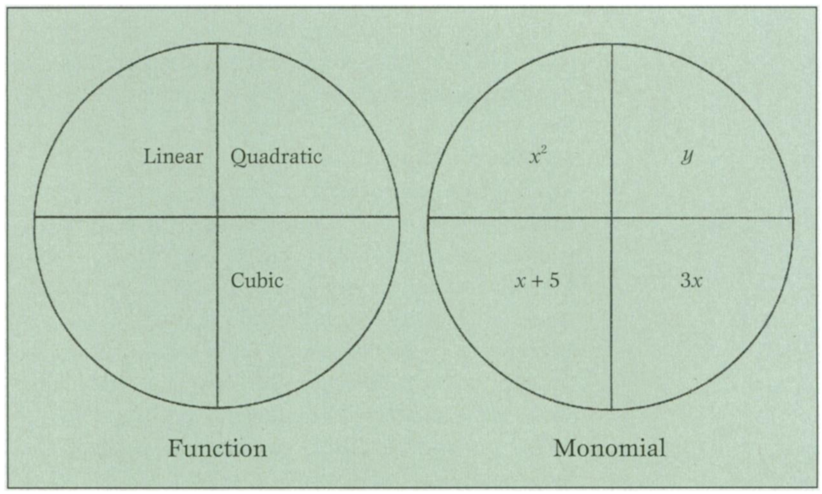

Fig. 6 Modified concept circles created by preservice teachers

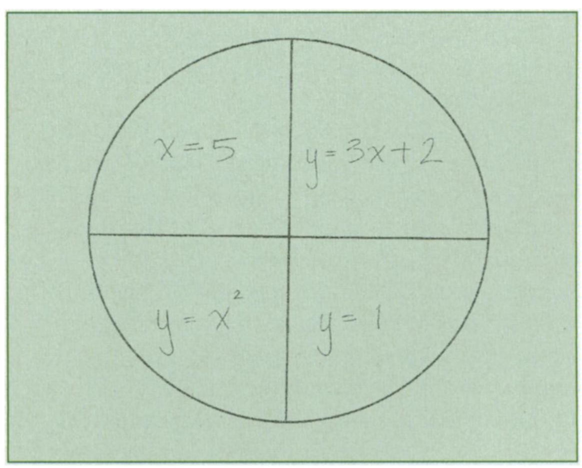

Fig. 7 Modified concept circle created by preservice teachers in which the expected response includes the concept and the nonexample; more than one response is possible

the nonexample would be $x + 5$. This concept circle helps students focus on the use of multiplication and exponentiation to create a monomial.

As an additional challenge, we do not initially provide the name or label of the circle, so the student has to name the concept and identify the section of the circle that contains the nonexample. Sometimes we present a modified concept circle that has more than one correct response. In these cases, the student must explain in what way one response is different from the others; the characteristics of each object become part of the discussion, thus prompting higher-level reasoning. One such concept circle is included as **figure 7**. For example, if $x = 5$ is the nonexample, the concept could be *functions* or *equations with graphs that intersect the y-axis*. If $y = x^2$ is the nonexample, the concept could be *linear equations* but not *linear functions*, because $x = 5$ does not express a functional relationship.

ANALOGIES

During this or another class period, the preservice teachers explore the use of analogies in mathematics. Research has shown that analogies can facilitate conceptual understanding and make concepts easier to remember (e.g., Halpern, Hansen, and Riefer 1990). Analogies can build connections between known and new ideas (Rubenstein 1996). In my class, work with analogies encourages creative thinking and places emphasis on conceptual understanding by focusing on relationships.

I begin with this example: *phrase : sentence :: expression :* ________. My class agrees that *equation* will correctly complete the analogy. Then I ask my preservice teachers to work in pairs to develop several analogies.

During this vocabulary activity, students use their mathematical reasoning skills to focus on key characteristics of concepts. Some of the preservice teachers' recent work is presented in **figure 8**. The first analogy has *reciprocal* as the unifying idea. In the second, the relationship is *inverse*. The third analogy uses a real-world setting to reinforce the meaning of mathematical terms. The last three emphasize similar ideas in the real world to focus on understanding the mathematical relationship or meaning. Although the quality and the level of mathematical accuracy of my students' analogies vary, I observe how difficult this task is for most of them, and I am pleased to see some creativity in their attempts to form connections across mathematics topics and between mathematics concepts and real-world applications.

cosine : secant :: sine : cosecant :: tangent : cotangent
$e^x : \ln x :: x^3 : \sqrt[3]{x}$
wall : room :: area : volume
grade : road :: slope : line
spoke : wheel :: radius : circle

Fig. 8 Analogies created by preservice teachers

CANCEL AND OTHER WORDS WITH MULTIPLE MEANINGS

Part of another class period is devoted to a discussion of the article "Why Cancel?" (Blubaugh 1988). In this article, Blubaugh described the various uses of the word *cancel* in mathematics. Because mathematics teachers are often comfortable with using this word in different settings, we can forget how confusing it can be to students to hear the same word applied to different operations. For example, *cancel* may refer to adding two opposites or subtracting two equal values, thus generating a sum or difference of zero. *Cancel* may also refer to dividing a nonzero number by itself when simplifying rational expressions. Teachers use *cancel* to describe completing operations used to solve equations and systems of equations. "Terms like this [*cancel*] mask the mathematical meanings we would like highlighted" (Rubenstein, personal communication, October 23, 2006). Reading this article and discussing it in class raises preservice teachers' awareness of how the same word can have different meanings and how multiple meanings can cause confusion for some students.

At times during the semester, there are opportunities to discuss other words with multiple meanings. Some words in mathematics change meanings when used as different parts of speech; two examples are *square* and *round*. In middle and high school classrooms, the verb *square* usually refers to multiplying a number by itself, while the noun *square* names a closed plane figure with four congruent sides and four right angles. As an adjective, *round* refers to a characteristic of some geometric shapes, yet as a verb *round* means to approximate a number to a specified place value.

Some words are used both in mathematics and in everyday English but have different meanings; examples include *expression* and *base*. Outside the classroom, some students will first think of facial expression when they hear *expression*, yet in mathematics the term refers to numerical and algebraic expressions. In geometry, *base* can refer to a side of a triangle or part of an expression that includes an exponent; however, students are more likely to be familiar with a base in baseball, a military base, the base of a column, or other everyday meanings of this word.

Still other words have a more specialized meaning in mathematics than they do in everyday English; an example is *similar*. Two shapes may look similar in that they resemble each other, but in mathematics certain criteria must be met for two figures to be similar (Barton and Heidema 2002). A good source with many more examples of everyday words with specialized mathematics meaning is Thompson and Rubenstein (2000).

CONCLUSION

These activities and the discussions that occur during methods class have indeed served to raise my preservice teachers' awareness of the critical role of mathematics vocabulary. They begin to see how important it is for them to use the correct word when describing a mathematical object; they see that it really does matter whether they say *line* or *segment*, *minus* or *negative*, and *expression* or *equation*. At the end of the semester, in a paper reflecting on their learning in the course, many preservice teachers describe new insights about vocabulary. One wrote, "We must understand that even though we know what we are talking about, all of the concepts are new to our students and must be explained very clearly and precisely." Another wrote, "I see the difference that simple vocabulary can have on a student's understanding of a particular concept."

In their future classrooms, I know that my preservice teachers will certainly make mistakes in vocabulary, but I have already observed them reminding themselves to say something besides *cancel* and correcting other vocabulary mistakes. Some have tried the vocabulary strategies, especially the graphic organizer shown in **figure 2**, with their classes during student teaching, and they gained some important insights into their own students' understandings and misunderstandings. These are good first steps that we hope will lead to better mathematics teaching and learning.

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